Inflation Policy Under Regime Instability

政治不稳定与通货膨胀政策选择

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Related Work

High inflation is often accompanied with the regime instability. Most analyses of inflation focus on

- relation between inflation and unemployment [Phillips,1958]
- relation between inflation and term structure [Cook and Hahn,1989]
- dynamic inconsistency of inflation [Kydland and Prescott,1977]

Some work about Regime Instability

- state capacity, conflict, and development [Besley and Persson,2010]
- state capacity in Latin America [Mauricio Cárdenas,2010]

Frameworks

- Total population is normalized to 1.Two groups, either of them comprises half of the total population.
- Two periods, $t \in \{1, 2\}$. Let I_t and O_t denote the incumbent and the opposition in period t with respectly.
- The incumbent decides the rate of inflation at the beginning of each period.
- *Rational expectation*.

Inflation and Utility

Inflation is an increase in the average price of the goods and services in terms of money.

- Inflation rate is considered as a policy chosen by the incumbent.
- Individual utility in each period is assumed to be linear.
- Individual time preference is β $(0 < \beta < 1)$, that's the discount rate.
- For group J_1, let W^{J_1} denote the life-time utility and $W_t^{J_1}$ denote the utility in period t. Thus the life-time utility of group J_1 is

$$W^{J_1} = W_1^{J_1} + \beta \mathbb{E}[W_2^{J_1}] \tag{1}$$

where $J_1 \in \{I_1, O_1\}$, and $\mathbb{E}$ means expectation taking.

Output and Inflation

- Aggregate demand disturbances have real effects and inflation expectation affects aggregate supply [Kydland and Prescott,1977].
- Output is determined by a Lucas-Rapping aggregate supply relation,

$$y_t = \bar{y}_t + b(\pi_t - \pi_t^e) \tag{2}$$

where y_t is output, $\bar{y}_t$ is the natural rate, and π_t and π_t^e are the actual and expected rates of inflation. Natural output grows at rate g, that means $\bar{y}_2 = (1+g)\bar{y}_1$.

Two Diff. Tax — Income Tax

The inflation affects the tax revenue in two ways. The first way of tax is called *income tax*, which is an explicit tax levied on everybody.

- The inflation will influence the total income, the actual output y_t.
- The tax rate is exogenously given. Take the average tax rate as τ_0.

Then the income tax revenue in period t is

$$\bar{T}_t = \tau_0 y_t \tag{3}$$

where τ_0 is constant. That means actual output and inflation have no effects on the average tax rate.

Two Diff. Tax — Seigniorage

The second way of tax is called *seigniorage*,revenue from government monetary expansion, referred to as seigniorage [Siegel,1981] .

- Seigniorage is really a tax on holders of money and government debt which is paid via inflation.
- Seigniorage revenues are often referred to as inflation-tax revenues.

It equals the increase in the nominal money stock per period divided by the price level:

$$\tilde{T}_t = \frac{\Delta M_t}{P_t} = \frac{\Delta M_t / M_t}{P_t / M_t} = g_M \frac{M_t}{P_t} \tag{4}$$

where g_M is the rate of money growth [Tanzi,1978]. (4) shows that real seigniorage equals the growth rate of the money stock times the quantity of real balances [Romer,1996].

Two Diff. Tax — Seigniorage(Continued)

- The real money demand depends negatively on the nominal interest rate and positively on real income, y_t.
- Assume that the real interest rate doesn't change in period 2, namely $r_1 = r_2 = \bar{r}$.

Since nominal interest rate is equal to the sum of real interest rate and the rate of inflation. We have

$$\frac{M_t}{P_t} = L(i_t, y_t) = L(\bar{r} + \pi_t, y_t) \qquad \text{where} \quad L_i < 0, L_Y > 0 \qquad (5)$$

Total Tax

If the economy is growing at a rate of growth $\tilde{g}$, some additional real balances will be demanded to meet that growth [Tanzi,1978]. We have $g_M = \pi_t + \tilde{g}$ in steady state. The total tax revenue in period t is

$$T_t = \bar{T}_t + \tilde{T}_t = \tau_0 y_t + (\pi_t + \tilde{g}) L(\bar{r} + \pi_t, y_t) \tag{6}$$

However, the incumbent has the right to redistribute the tax revenues that means the real tax rate for the incumbent is negative.

Stability of Regime

- Inflation will influence the probability that the incumbent stays in power in next period.
- Probability for the incumbent I_1 to stay in power is $\lambda(\pi_1)$.
- The marginal cost of inflation decreases as the inflation rate increases.

That means

$$\mathbf{P}\{\text{stay in power}\} = \lambda(\pi_1) \quad \text{where } \lambda' < 0,\ \lambda'' > 0 \tag{7}$$

Stability of Regime(Continued)

- λ stands for the stability of regime or the political risk of inflation.
- This can be thought as a reduced form of some political process. Whoever wins will become the new incumbent.
- Let $\bar{\pi}_t$ denote the natural rate of inflation in economy in period t (where $t = 1$ or $t = 2$), if $\pi_t = \bar{\pi}_t$, then $\lambda(\pi_t) = \lambda_0$.
- The parameter satisfies the condition: $0 < \lambda_0 < 1$.

Adjustment Cost

- Adjustment cost of inflation deviation includes the social unrest cost , shoe-leather cost, menu cost and etc.
- Aassume that inflation deviation is costly, and the marginal cost of inflation increases as inflation deviation rises.

A simple way to capture these assumptions is to make adjustment cost quadratic in inflation [Romer,1996]. The adjustment cost is

$$\bar{C}_t = \frac{1}{2}a(\pi_t - \bar{\pi}_t)^2 \quad \text{where } a > 0 \tag{8}$$

Adjustment cost is considered as a negative public goods.

Administration Cost

Except for the adjustment cost, the incumbent have to regulate the economy and the society. Denote the administration cost $\tilde{C}$.

We would like to make administration cost quadratic in inflation as well. The administration cost is

$$\tilde{C}_t = \frac{1}{2}\alpha(\pi_t - \bar{\pi}_t)^2 \quad \text{where } \alpha > 0 \tag{9}$$

Some Remarks On ACs

- Both the adjustment cost and the administration cost are computed by group, not by capita.
- The parameters a and α reflect the relative importance of inflation and output in utility function.
- Inflation deviate further from the natural rate of inflation $\bar{\pi}_t$, the greater the adjustment cost and administration cost will be.
- Assume that $\bar{\pi}_t = 0$. Then the adjustment cost of inflation are $\bar{C}_t = a/2\pi_t^2$, and the administration cost of inflation is $\tilde{C}_t = \alpha/2\pi_t^2$.

Timing

Just as **Figure**1 shows, we explain the timing in detail.

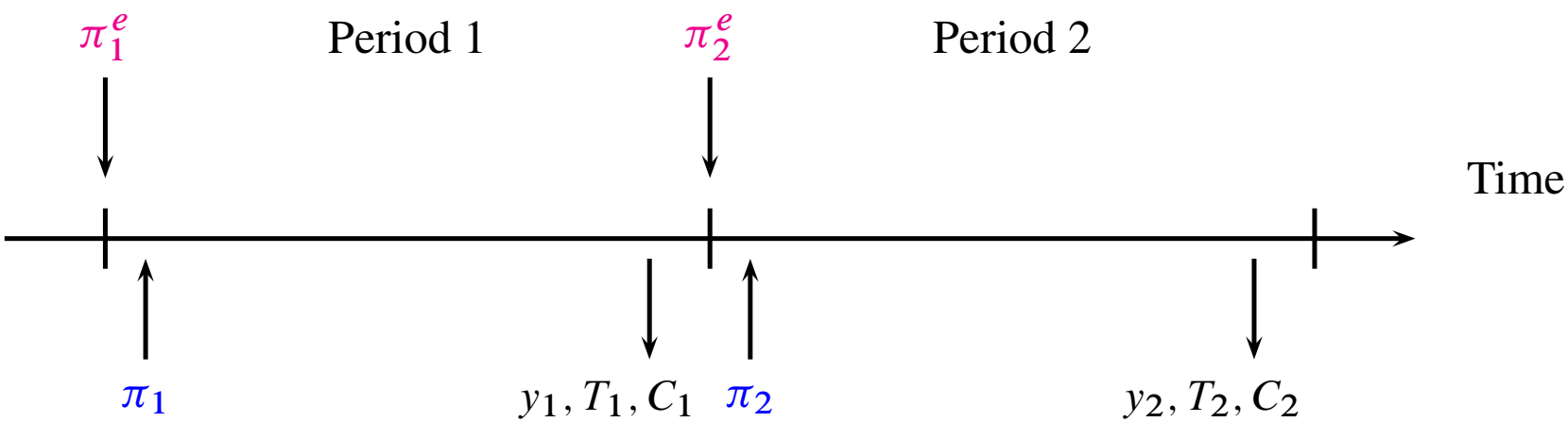

Figure 1: Timing

Preparation

- Since we have considered two periods, we cannot choose the policy choices separately.
- To study the utility in period t, we let W_t^I and W_t^O denote the utility of the incumbent and the opposition in period t with respectly, where $t = 1$ or $t = 2$.

The Utility Of The Incumbent

The incumbent government can redistribute the tax revenue. And everybody suffers from the adjustment cost. With everything considered the utility of the incumbent in period t (where $t = 1, 2$) is

$$W_t^I = \frac{1}{2} y_t + \frac{1}{2} T_t - \bar{C}_t - \tilde{C}_t \tag{10}$$

where (10) uses the fact that each group comprises half the total population, each group suffers from the social unrest and other costs(in sum, the adjustment cost).

The Utility Of The Opposition

Similarly, the utility of the opposition in period t (where $t = 1, 2$) is

$$W_t^O = \frac{1}{2} y_t - \frac{1}{2} T_t - \bar{C}_t \tag{11}$$

The difference between the utility of the incumbent and the opposition is the tax and the administration cost.

- For the opposition, they have no rights to redistribute the tax revenue, thus the effects of tax on their income are negative.
- The incumbent have to regulate the economy and society. The administration cost is $\tilde{C}_t$, but for the opposition, they have no obligation to pay such an expense.

Life-time Utility

Now, we check the total utility of the incumbent I_1 and the opposition O_1.

- Owing to the regime stability, we consider the expectation of utility.
- I_1 will stay in power with probability $\lambda(\pi_1)$.
- I_1 will become the opposition in period 2 with probability $1-\lambda(\pi_1)$.

Using equation (1), the life-time utility of the incumbent in period 1 (I_1) is

$$W^{I_1} = W_1^I + \beta\left\{\lambda(\pi_1)W_2^I + (1-\lambda(\pi_1))W_2^O\right\} \tag{12}$$

Similarly, the life-time utility of the opposition in period 1 (O_1) is

$$W^{O_1} = W_1^O + \beta\left\{(1-\lambda(\pi_1))W_2^I + \lambda(\pi_1)W_2^O\right\} \tag{13}$$

Mechanism

Figure2 shows the mechanism

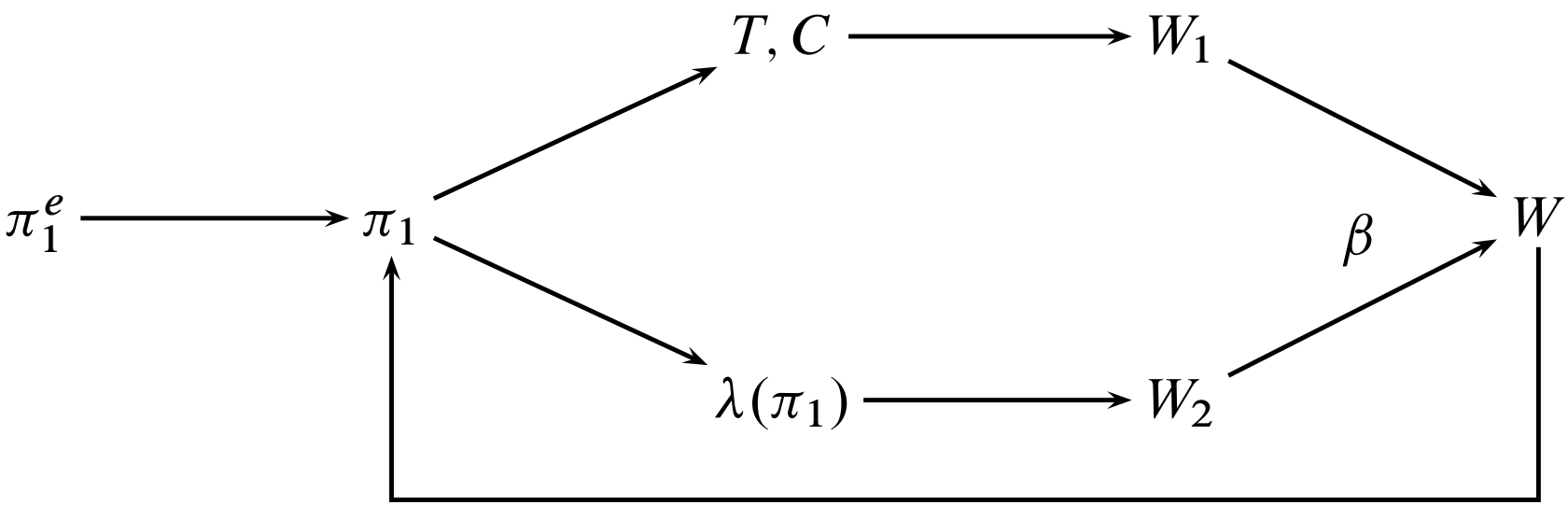

Figure 2: Mechanism

Equilibrium Condition and State

For the moment, we focus on equilibrium states[1].

- The incumbent face the tradeoff, politcal risks and benefits.
- The incumbent will choose a positive level of inflation.
- From the Lucas supply curve,(2), we know that at equilibrium state, $y_t^* = \bar{y}_t$, that means $\tilde{g} = g$, $y_2^* = (1+g)y_1^*$. If we let $\bar{y} = \bar{y}_1$ then we have $y_2^* = (1+g)\bar{y}$ and $y_1^* = \bar{y}$.

Proposition 1

At the equilibrium state, the actual inflation rate is equal to the inflation anticipation, the real output equals with the natural output, namely $\pi_t^* = \pi_t^e > 0$, $y_t^* = \bar{y}_t$. The growth rate of actual output $\tilde{g} = g$.

[1] we asterisk the variables to signify the variables in equilibrium state

Money Demand

- Real output and interest rate are unaffected
- The real economic variables will not be affected by inflation.

Frequently, we use the money demand function proposed by Cagan[Cagan,1956]. For simplicity, consider the money-demand function given by

$$\frac{M_t}{P_t} = \mu - \nu i_t + y_t \qquad \nu > 0 \tag{14}$$

where ν is assumed to satisfy $\bar{y} > \nu$.

- According to the money demand function ,the real demand for money is $L(\bar{r} + \pi_t, y_t) = \mu - \nu i_t + y_t$.
- If $\pi_t = \bar{\pi}_t = 0$, then natural money demand is $\bar{m}_t = \mu - \nu \bar{r} + \bar{y}_t$.

Optimal Policy In Period 2

Under the steady state, we solve the utility maximization problem(get the optimal inflation policy in period 2,π_2^*).
Take partial derivatives of π_2^* with respect to a, α, g and $\bar{y}$. We have

$$\frac{\partial \pi_2^*}{\partial a} = \frac{\partial \pi_2^*}{\partial \alpha} < 0 \quad \frac{\partial \pi_2^*}{\partial g} > 0, \quad \frac{\partial \pi_2^*}{\partial \bar{y}} > 0 \tag{15}$$

Proposition

From the derivatives above, we have the following proposition

Proposition 2

The incumbent government in period 2 will select the **unique** rate of inflation. The equilibrium inflation rate is positively related to the natural growth rate of output g as well as the output $\bar{y}$. However, it's negatively relevant to the parameters in adjustment and administration cost function, namely , a and α.

Proposition 2 illustrates that

- the optimal rate of the inflation rises when the growth rate of the economy or the development of the country increases.
- deviation from the natural state is costly. If a or α increases, then the optimal inflation rate will dereases.

Utility Gap

Consider the difference of utility (**utility gap**) between the incumbent and the opposition in period t

$$\Delta W_t = W_t^I - W_t^O \tag{16}$$

Differential the utility gap function with respect to a, α, g and $\bar{y}$. Substitute π_2^* into the utility gap function yields

$$\frac{\partial \Delta W_2^*}{\partial \alpha} < \frac{\partial \Delta W_2^*}{\partial a} < 0, \qquad \frac{\partial \Delta W_2^*}{\partial g} > 0, \qquad \frac{\partial \Delta W_2^*}{\partial \bar{y}} > 0 \tag{17}$$

Proposition

From (17) we have the following proposition

Proposition 3

At equilibrium, higher growth rate of economy (g) and the natural output ($\bar{y}$) will broaden the utility gap between the incumbent and the opposition in period 2. On the contrary, larger parameters in cost function(namely a and α) will narrow the utility gap.

This proposition conveys an idea that

- If the government gains more benefits from holding power, then the utility gap will be broadened.
- It will enlarge the utility gap and aggravate the inequality.

Optimal Policy In Period 1 I

Now we turn to the inflation rate selection in period 1. For the incumbent government I_1, he faces the tradeoffs.

- The probability to stay in power decreases as he seeks for higher payoff in period 1.
- The utility in period 2 is uncertain and needs to be discounted.
- If I_1 stays in power, then he can redistribute the tax revenues.

In this problem, we have to use the backward induction method. The incumbent in period 1 seeks the optimal inflation rate in period 1 to maximize the life-time utility of his own group,

$$\max_{\pi_1} \quad W^{I_1} = W_1^{I_1} + \beta \left\{ \lambda(\pi_1) W_2^I \Big|_{\pi_2=\pi_2^*} + (1-\lambda(\pi_1)) W_2^O \Big|_{\pi_2=\pi_2^*} \right\} \tag{18}$$

Optimal Policy In Period 1 II

We consider a special case. Since we cannot get the explicit expression of π_1^*, we cannot tell how large it is. But we know that the optimal inflation rate in period 1 exists and the value is positive. The critical condition is

$$\lambda''(\pi_1^*) < \frac{a + \alpha + \nu}{\beta \Delta W_2^*} \tag{19}$$

From (17), we know that $\partial \Delta W_2^* / \partial g < 0$ and $\partial \Delta W_2^* / \partial \bar{y} < 0$. Since $\lambda'(\pi_1^*) < 0$, if the optimal inflation rate satisfies the condition (19), we have

$$\frac{\partial \pi_1^*}{\partial g} < 0 \qquad \frac{\partial \pi_1^*}{\partial \bar{y}} < 0 \tag{20}$$

Proposition

Proposition 4

If the optimal inflation rate satisfies the condition (19), then the equilibrium inflation rate in period 1 is negatively relevant to the growth rate of economy (g) as well as the natural output ($\bar{y}$).

g & $\bar{y}$ stand for the development of the economy (country).

- Under certain condition, the inflation rate is low in more developed countries (stay in power to redistribute).
- Higher inflation will enlarge the high political risk.

That's exactly what happened in reality. In a democratic rich country, the inflation will be controlled to stay low level.

Inflation and Regime Stability

At last, we check the relation between the stability of regime and the optimal inflation rate. The elasticity of regime stability with respect to inflation is given by

$$\varepsilon = \frac{(\ln \lambda)'}{(\ln \pi_1)'} = \lambda' \frac{\pi_1}{\lambda} \tag{21}$$

We want to check whether $\partial \pi_1 / \partial \varepsilon$ is positive or negative. Since

$$\frac{\partial \pi_1}{\partial \varepsilon} = \frac{1}{\partial \varepsilon / \partial \pi_1} \tag{22}$$

Then it's equivalent to check whether $\partial \varepsilon / \partial \pi_1$ is positive or negative

$$\frac{\partial \varepsilon}{\partial \pi_1} = \frac{\lambda'}{\lambda} - \frac{\pi_1}{\lambda^2} \left((\lambda')^2 - \lambda \lambda'' \right) \tag{23}$$

Proposition

If the probability function has the property that $(\lambda')^2 - \lambda\lambda'' > 0$, then $\partial\varepsilon/\partial\pi_1$ is negative.

Proposition 5

If the probability function $\lambda(\cdot)$ satisfies $(\lambda')^2 - \lambda\lambda'' > 0$ at equilibrium, then higher inflation elasticity of regime stability will result in lower inflation selection of the incumbent in period 1.

- Under certain conditions, higher political cost of inflation will lower the rate of inflation at EQ.
- If the inflation cost is high, I_1 has to give up much utility in period 2.
- I_1 will lower the inflation expecting higher expected utility in period 2.

Conclusion

- Rational expectation gives the results in proposition 1. The expectation for inflation equals with the actual inflation.
- In period 2, the incumbent doesn't need to consider the next period's utility, hence the inflation rate will be relatively high.
- In period 1, the incumbent has to consider the stability of regime.
- At the equilibrium, higher growth rate of economy and output broaden the utility gap in period 2.
- Larger parameters in cost function will narrow the utility gap.
- Regime instability, the most important factor, influences the expected utility in period largely.
- Higher inflation elasticity of regime stability will result in lower inflation rate.

Thanks

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Thank you for your attention!