INFLATION POLICY UNDER REGIME INSTABILITY

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Abstract: This paper develops a two-period model of inflation policy selection with regime instability. The incumbent government can redistribute the tax revenue. The stability of regime rests with the rate of inflation. With public rational expectation, the government's optimal inflation rate equals with peoples's expectation. It also argues that the growth rate of output and the development of economy positively affect the inflation level selection in period 2, while in period 1, the relation is quite contrary. The cost of deviation from the natural inflation rate is negatively relevant to the inflation selection. Higher growth rate of economy and output will broaden the utility gap in period 2. Higher inflation elasticity of regime stability will result in lower inflation which is selected by the incumbent in period 1. The probability influences the inflation policy selection in period 1 to a great extent.

Keywords: Inflation regime instability, inflation elasticity of regime stability

High inflation is often accompanied with the regime instability. Most analyses of inflation focus on the relation between inflation and term structure [Cook and Hahn, 1989], the relation between inflation and unemployment [Phillips, 1958], inflation targeting [Svensson, 1997], and dynamic inconsistency of inflation [Kydland and Prescott, 1977].

In this paper, we will turn to the *New Political Economy*. The aim of the new political economy is to understand important issues that arise in the policy sphere. It is not an effort by economists to colonize political science. Rather, the main concern is to extend the competence to analyze the issues that require some facility with economic and political decision making [Besley, 2004]. The new political economy perspective breathes life into the state as an autonomous actor with it's perspective, from all other actors in society [Kirshner, 2001]. Influenced greatly by the work of North [North, 1981] and Levi(1988) here the state is conceptualized as an egoistic rent seeking maximizer. We extend to two utilitarian group seek after their own group utility maximization.

Timothy Besley and Torsten Persson analysed the state capacity from the new political economy sphere [Besley and Persson, 2010]. We will extend this method to analyze the inflation selection, the regime stability and economic development.

This paper puts forward a simple model of utilitarian government in two-period economy. The opposition has to pay the tax and the incumbent government has the right to redistribute the tax revenue. Both of them have to pay the cost of inflation deviated from the natural output. Owing to the instability of regime, the government in period 1 may be the opposition in

period 2. The incumbent in period 1 consider the life-time utility of his own group. That's the background of the inflation policy selection problem under regime instability.

Part 1 illustrates the basic setup of our model. We extend the new political economic method in our analysis. We also assume there two groups in the economy, one is the incumbent and the other is the opposition. Besides, the public have their expectation for inflation with perfect information and rational expectation.

From the utilitarian group, the utility in period 2 in less valuable, thus the utility in period 2 have to be discounted when they consider their life-time utility. Moreover, the utility in period 2 is uncertain, thus the groups consider the utility expectation. The relationship between the actual output and the rate of inflation is given by the Lucas supply curve.

When we consider the benefits of inflation for the incumbent, we come to the two different taxes, the income tax and the seigniorage. The income tax is levied on everybody and the seigniorage is levied on the money holder. We will analyze the seigniorage in detail with the growth rate of money stock and the quantity of real balances considered. Something we should bear in mind is that the incumbent has the right to redistribute the tax, thus the real tax rate for the incumbent is negative.

When considering the cost of inflation, we need to consider two main aspects. The first is the regime instability causing by inflation, it's the explicit cost of inflation. The other is the adjustment cost and the administration cost. Once the inflation in period 1 is determined, the probability that the incumbent stay in power in period 2 is determined. That means the stability of regime is realized. The probability will affect the right to redistribute the tax revenues, hence it will affect the utility of both group in period 2. The other cost is the adjustment cost and the administration cost, the former includes the shoe-leather cost, the menu cost and the social unrest cost. The latter includes the cost of regulation and infrastructure.

In Part 2, we analyze the equilibrium state of the economy. We model the utility of the incumbent in period 2 and the life-time utility of the incumbent and the opposition in period 1. At equilibrium, the rational expectation theory requires that the equivalence relation between the inflation anticipation and the actual inflation. To compute the optimal inflation rate in period 1, we use the backward induction method.

Here we leave out the complicated computational problem, we analyze the optimal inflation rate selection through comparative static analysis. We have some views to point out.

Rational expectation requires that the expectation for inflation equals with the actual inflation. People adjust their expectation if it deviates from the policy. It is a game under perfect information. At equilibrium, the actual output equals with the natural output, the growth rate of actual output is equal to the natural growth rate, and we will illustrate it in detail in the context.

With equilibrium condition satisfied, we want analyze the relationship between the inflation

selection in two periods and the growth rate of output and the natural output.

In period 2, the incumbent doesn't have to consider the next period's utility, hence the inflation rate will be relatively high. But in period 1, the incumbent have to consider the stability of regime. If the future utility is promising, the incumbent will be more willing to stay in power in period 2. On the contrary, if the utility of the incumbent in period 2 is small, for example, the economy is less developed, then the rate of inflation in period 1 will be relatively large.

The next problem is the utility gap between the incumbent and the opposition. Under the condition of equilibrium, higher growth rate of economy and output result in more benefits from holding in power, it may broaden the utility gap in period 2. On the contrary, larger parameters in cost function may narrow the utility gap.

At last we will analyze the influence of the probability function that the incumbent in period stays in power in period 2. It will influence the expected utility in period largely. We expect that higher inflation elasticity of regime stability will result in lower inflation which is selected by the incumbent in period 1. Since the political cost of inflation is large.

1. Basic Model setup

1.1. *Frameworks*

The model is stripped down to give a simple analysis of the inflation policy. Total population size is normalized to 1. There are two groups, group A and group B, either of them comprises half of the total population. At the beginning, we consider two time periods, $t \in \{1, 2\}$.

Let I_t denotes the incumbent government in period t and O_t denotes the other group(the opposition). Besides, the incumbent government has the right to decide the rate of inflation. At the beginning of each period, the incumbent government chooses the inflation level, but the opposition has to face the reality. In brief, the incumbent has the initiative, and the opposition is passive.

Another important assumption is people's expectation for inflation is rational expectation rather than the adaptive expectation. That means the information is perfect. The incumbent and the public have the same knowledge.

1.2. *Inflation and Utility*

Inflation is an increase in the average price of the goods and services in terms of money. But here we do not examine the market for money. The main point is that inflation level is considered as a policy chosen by the incumbent government. We don't care how the money market affects the inflation in detail.

Whether the incumbent choose low inflation rate or high inflation rate depends on his utility in two periods. And for simplicity, individual utility in each period is assumed to be linear.

Given the value of time, the utility in period 2 needs to be discounted. Suppose the individual time preference is β, since the utility in next period is less valuable, $0 < \beta < 1$.

The objectives of the incumbent and the opposition are to maximize their own group utility. For group J_1, let W^{J_1} denote the life-time utility and $W_1^{J_1}$ denote the utility in period 1. Since the incumbent government of period 2 is undetermined in period 1. Thus the life-time utility of group J_1 is

$$W^{J_1} = W_1^{J_1} + \beta \mathbb{E}[W_2^{J_1}] \tag{1}$$

where $J_1 \in \{I_1, O_1\}$, and $\mathbb{E}$ means expectation taking.

1.3. *Output and Inflation*

In this section, we come to inflation effects on income. Kydland and Prescott considered an economy where aggregate demand disturbances have real effects and expectations concerning inflation affect aggregate supply [Kydland and Prescott, 1977]. Output is determined by a Lucas-Rapping aggregate supply relation,

$$y_t = \bar{y}_t + b(\pi_t - \pi_t^e) \tag{2}$$

where y_t is output, $\bar{y}_t$ is the natural rate, and π_t and π_t^e are the actual and expected rates of inflation. Natural output grows at rate g, that means $\bar{y}_2 = (1 + g)\bar{y}_1$. If the growth rate of output is high enough, we suppose that the incumbent government will be inclined to stay in power.

1.4. *Two Different Tax: Income Tax and Seigniorage*

The inflation affects the tax revenue in two forms. The first tax called *income tax*, which is an explicit tax levied on everybody. The second form of tax is called *seigniorage*,revenue from government monetary expansion, referred to as seigniorage [Siegel, 1981] . Seigniorage is an implicit tax.

From the former analysis, we know the inflation will influence the total income, namely the actual output y_t. If the tax rate doesn't change, then the total tax revenue linearly depend on the total income. Take the general average tax rate as τ_0, then the explicit tax revenue in period t is

$$\bar{T}_t = \tau_0 y_t \tag{3}$$

where τ_0 is constant which means actual output has no effects on the average tax rate.

Seigniorage is really a tax on holders of money and government debt which is paid via inflation. Seigniorage revenues are often referred to as inflation-tax revenues. It equals the increase in the nominal money stock per period divided by the price level:

$$\tilde{T}_t = \frac{\Delta M_t}{P_t} = \frac{\Delta M_t / M_t}{P_t / M_t} = g_M \frac{M_t}{P_t} \tag{4}$$

where g_M is the rate of money growth [Tanzi, 1978]. (4) shows that real seigniorage equals the growth rate of the money stock times the quantity of real balances [Romer, 1996].

Assume that the real money demand depends negatively on the nominal interest rate and positively on real income, y_t. We assume that the real interest rate doesn't change in period 2, namely $r_1 = r_2 = \bar{r}$. Since nominal interest rate is equal to the sum of real interest rate and the rate of inflation. We have

$$\frac{M_t}{P_t} = L(i_t, y_t) = L(\bar{r} + \pi_t, y_t) \qquad \text{where} \quad L_i < 0, L_Y > 0 \tag{5}$$

If the economy is growing at a rate of growth g, some additional real balances will be demanded to meet that growth [Tanzi, 1978]. In our model the growth rate of actual output is $\tilde{g}$, we have $g_M = \pi_t + \tilde{g}$ in steady state. Using (3), (4) and the demand of real money,(5), the total tax revenue in period t is

$$T_t = \bar{T}_t + \tilde{T}_t = \tau_0 y_t + (\pi_t + \tilde{g}) L(\bar{r} + \pi_t, y_t) \tag{6}$$

However, the incumbent has the right to redistribute the tax revenues that means the real tax rate for the incumbent is negative. The tax revenues will be divided equally in the incumbent group.

1.5. *Stability of Regime*

In period 1, I_1 is in power, suppose power can be peacefully transferred to the opposition, which happens with probability λ. Since the current rate of inflation will influence the probability that the incumbent government stay in power (namely $I_1 = I_2$), hence λ is relevant to the rate of inflation in period 1, π_1, that means the probability for the incumbent government to stay in power is $\lambda(\pi_1)$.

If the rate of inflation, π_1, increases, the probability that I_1 stay in power decreases in intuitionally. And as π_1 increases, the probability declines less rapidly. Equivalently, the marginal "cost" of inflation decreases as the inflation rate increases. That means

$$\mathbf{P}\{\text{stability of regime}\} = \lambda(\pi_1) \quad \text{where } \lambda' < 0,\ \lambda'' > 0 \tag{7}$$

λ stands for the stability of regime. This can be thought as a reduced form of some political process. As a result, whoever wins will become the new incumbent. It's the political cost (or political risk) of inflation.

Let $\bar{\pi}_t$ denote the natural rate of inflation in economy in period t (where $t = 1$ or $t = 2$), if $\pi_t = \bar{\pi}_t$, then $\lambda(\pi_t) = \lambda_0$. The parameter satisfies the condition: $0 < \lambda_0 < 1$.

1.6. *Adjustment Cost and Administration Cost*

The next problem is the adjustment cost of inflation deviation, which includes the social unrest cost , shoe-leather cost, menu cost and etc. We assume that inflation deviate from the natural rate of inflation , $\bar{\pi}_t$, is costly, and the marginal cost of inflation increases as inflation deviation rises. A simple way to capture these assumptions is to make adjustment cost quadratic in inflation [Romer, 1996]. The adjustment cost is

$$\bar{C}_t = \frac{1}{2}a(\pi_t - \bar{\pi}_t)^2 \quad \text{where } a > 0 \tag{8}$$

In our model, adjustment cost is considered as a negative public goods that both of the incumbent government and the opposition have to "consume".

Except for the adjustment cost, the incumbent have to regulate the economy and the society. For instance, the incumbent has to build the court system and the policy station. That's an expense for the incumbent. Denote the administration cost $\tilde{C}$. It is similar with the adjustment cost, we would like to make administration cost quadratic in inflation as well. The administration cost is

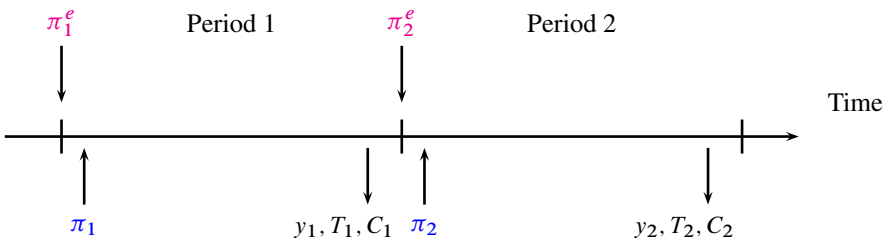
$$\tilde{C}_t = \frac{1}{2}\alpha(\pi_t - \bar{\pi}_t)^2 \quad \text{where } \alpha > 0 \tag{9}$$

Both the adjustment cost and the administration cost are counted by group, not by capita. The parameters a and α reflect the relative importance of inflation and output in utility function. From the experience of life, parameters a and α may be relevant to the natural output $\bar{y}$. From the adjustment cost equation (8) and administration cost equation (9), we conclude that when inflation deviate further from the natural rate of inflation $\bar{\pi}_t$, the greater the adjustment cost and administration cost will be. To concentrate on the inflation policy, we simplify the model with an assumption $\bar{\pi}_t = 0$. Then the adjustment cost of inflation are $\bar{C}_t = a/2\pi_t^2$, and the administration cost of inflation is $\tilde{C}_t = \alpha/2\pi_t^2$.

1.7. *Timing*

Just as **Fig** 1.1 shows, we explain the timing in detail.

Fig 1.1: Timing



- The incumbent government in period 1 is exogenously given, the expectation for the rate of inflation in period 1 is determined with rational expectation. Suppose the realized incumbent in period 1 is I_1, the opposition in period 1 is O_1. People's expectation for inflation in period 1 is π_1^e.

- The incumbent government choose the level of inflation π_1 in period 1. The policy is executed at the beginning of period 1 till the end of the period.

- The probability of I_1 to stay in power $\lambda(\pi_1)$ is determined by the rate of inflation in period 1. Namely, the stability of regime is determined.

- The output y_1, the total tax revenue T_1 and adjustment cost $\bar{C}_1$ and administration cost $\tilde{C}_1$ in period 1 are realized at the end of period 1. People's expectation for inflation of the next period π_2^e takes shape.

- The incumbent government in period 2 is realized, the incumbent in period 2 chooses the level of inflation,π_2, given the expectation for inflation in period 2, π_2^e.

- The output y_2, the total tax revenue T_2 and adjustment cost $\bar{C}_2$ and administration cost $\tilde{C}_2$ in period 2 are realized at the end of period 2.

2. Optimal policy

2.1. *Utility Maximization Problem*

We begin with choices in step 2, since we have considered the value of time. Thus we cannot choose the policy choices separately.

To focus on the policy selection in each period, we develop a theory of structuralization. To study the utility in period t, we let W_t^I denote the utility of the incumbent in period t, and W_t^O denote the utility of the opposition in period t, where $t = 1$ or $t = 2$.

The incumbent government can redistribute the tax revenue. In our model the public goods is absent. Hence the redistribution can be interpreted as a negative tax "levied" on the incumbent. And everybody suffers from the adjustment cost. With everything considered the utility of the incumbent in period t (where $t = 1, 2$) is

$$W_t^I = \frac{1}{2}y_t + \frac{1}{2}T_t - \bar{C}_t - \tilde{C}_t \tag{10}$$

$$= \frac{1}{2}y_t + \frac{1}{2}[\tau_0 y_t + (\pi_t + \tilde{g})L(\bar{r} + \pi_t, y_t)] - \frac{1}{2}(a + \alpha)\pi_t^2 \tag{11}$$

$$= \frac{1}{2}\left\{\big((1 + \tau_0)[\bar{y}_t + b(\pi_t - \pi_t^e)] + (\pi_t + \tilde{g})L(\bar{r} + \pi_t, y_t)\big) - (a + \alpha)\pi_t^2\right\} \tag{12}$$

where (10) uses the fact that each group comprises half the total population, each group suffer from the social unrest and other costs(in sum, the adjustment cost). And (11) uses the sum of income tax and seigniorage (6) and the adjustment cost equation (8) and the administration cost equation (9). The last equivalent relation comes from the aggregate supply equation (2) (Lucas supply curve) .

Similarly, the utility of the opposition in period t (where $t = 1, 2$) is

$$W_t^O = \frac{1}{2}y_t - \frac{1}{2}T_t - \bar{C}_t \tag{13}$$

$$= \frac{1}{2}y_t - \frac{1}{2}[\tau_0 y_t + (\pi_t + \tilde{g})L(\bar{r} + \pi_t, y_t)] - \frac{1}{2}a\pi_t^2 \tag{14}$$

$$= \frac{1}{2}\left\{(1-\tau_0)\left[\bar{y}_t + b(\pi_t - \pi_t^e)\right] - (\pi_t + \tilde{g})L(\bar{r} + \pi_t, y_t) - a\pi_t^2\right\} \tag{15}$$

The difference between the utility of the incumbent and the opposition is the tax and the administration cost. For the incumbent, the tax is a positive "income", but for the opposition, they have no rights to redistribute the tax revenue, thus the effects of tax on their income are negative . Besides, the incumbent have to regulate the economy and society. The administration cost is $\tilde{C}_t$, but for the opposition, they have no obligation to pay such an expense.

Now, we check the total utility of the incumbent I_1 and the opposition O_1. Owing to the regime stability, the utility in period 2 for both the incumbent and the opposition are uncertain. To study the utility of them, we consider the expectation of utility in period 2.

At first, consider the incumbent in period 1, I_1 will stay in power with probability $\lambda(\pi_1)$, and I_1 will become the opposition in period 2 with probability $1 - \lambda(\pi_1)$. Using equation (1), the life-time utility of the incumbent in period 1(namely I_1) is

$$W^{I_1} = W_1^I + \beta\left\{\lambda(\pi_1)W_2^I + (1-\lambda(\pi_1))W_2^O\right\} \tag{16}$$

Similarly, the life-time utility of the opposition in period 1 (namely O_1)is

$$W^{O_1} = W_1^O + \beta\left\{(1-\lambda(\pi_1))W_2^I + \lambda(\pi_1)W_2^O\right\} \tag{17}$$

2.2. *Equilibrium Analysis*

For the moment, we focus on equilibrium states[1]. We need to check what the policy will the incumbent in period 1, I_1, choose under the condition that he knows the inflation anticipation in period 2. In other words, what's the best response function $g(\cdot)$,where $\pi_2 = g(\pi_2^e)$, for I_1 if I_1 knows the expectation for inflation is π_2^e. Since the best response function is also "predictable", people will adjust their expectation to narrow the error of anticipation. It just likes a two period game.

[1]we asterisk the variables to signify the variables in equilibrium state

Under the theory of rational expectation, it's easy to check the public will adjust the expectation until the actual inflation rate and the expected inflation are equal, $\pi_2 = \pi_2^e$. That's the equilibrium condition.

Similarly, the equilibrium in period 1 also requires the equivalence of actual inflation and inflation anticipation. The incumbent face the tradeoff, on the one hand, the incumbent have to undertake the cost of adjustment and administration, on the other hand, the incumbent gains the tax benefits. Generally, the incumbent will choose a positive level of inflation. From (6), we know if the rate of inflation is negative(actually it means $\pi_t - \bar{\pi}_t < 0$) and sufficiently small ($\pi_t + g < 0$), then the incumbent government cann't get any seigniorage. Moreover, the incumbent have to "pay" the cost of adjustment and administration. Thus the optimal rate of inflation should be equal or greater than the natural rate of inflation $\bar{\pi}_t$, which we have assumed to be 0.

From the Lucas supply curve,(2), we know that at equilibrium state, $y_t = \bar{y}_t$, since the natural output grows at rate g, that means $\tilde{g} = g$, to wit, $y_2 = (1+g)y_1$. If we let $\bar{y}$ denote the natural output in period 1, then we have $y_2 = (1+g)\bar{y}$ and $y_1 = \bar{y}$.

Proposition 1: *At the equilibrium state, the actual inflation is equal to the inflation anticipation, the real output equals with the natural output, namely* $\pi_t = \pi_t^e = \pi_t^* > 0$ *(*$t = 1$ *or* $t = 2$*),* $y_2 = \bar{y}_2, y_1 = \bar{y}_1$. *The growth rate of actual output* $\tilde{g} = g$.

It's reasonable to think that output and real interest rate are unaffected by the rate of money growth, the rational expectation tells us that the real economic variables in the economy will not be affected by inflation. As an example of the relation between inflation and steady-state seigniorage, consider the money-demand function given by

$$\frac{M_t}{P_t} = \mu - \nu i_t + y_t \qquad \nu > 0 \tag{18}$$

where ν is assumed to satisfy $\bar{y} > \nu$. Frequently, we use the money demand function proposed by Cagan[Cagan, 1956]. According to the money demand function given by (18), the real demand for money is $L(\bar{r} + \pi_t, y_t) = \mu - \nu i_t + y_t$. If $\pi_t = \bar{\pi}_t = 0$, then natural money demand is $\bar{m}_t = \mu - \nu\bar{r} + \bar{y}_t$. Substitute the real demand for money into (6), then we can get the total tax revenues. And if we substitute the real demand for money into (12) and (15), we will get the utility of the incumbent and the opposition in period t (where $t = 1$ or $t = 2$).

From now on, we just consider the Equilibrium State, where $\pi_t = \pi_t^e = \pi_t^* > 0$. The last relation comes from the fact that if the It's still unknown what the policy (inflation rate) the incumbent will choose. Under the steady state, the utility of the incumbent becomes

$$W_t^I = \frac{1}{2}\left\{(1+\tau_0)\bar{y}_t + (\pi_t + g)(\bar{m}_t - \nu\pi_t) - (a+\alpha)\pi_t^2\right\} \tag{19}$$

Similarly, the utility of the opposition is

$$W_t^O = \frac{1}{2}\{(1-\tau_0)\bar{y}_t - (\pi_t + g)(\bar{m}_t - \nu\pi_t) - a\pi_t^2\} \tag{20}$$

Take derivatives of the two sides with respect to π_t,

$$\frac{\partial W_t^I}{\partial \pi_t} = \frac{1}{2}\{\bar{m}_t - g\nu - 2(a+\alpha+\nu)\pi_t\} \tag{21}$$

Since the rate of inflation π_2^* maximize the utility of the incumbent, $(\partial W_t^I/\partial\pi_t)\big|_{\pi_2^*} = 0$. Rearrange the first-order condition for the incumbent inflation rate π_t^* with the maximization condition yields

$$\text{F.O.C}: \qquad \pi_2^* = \frac{1}{2}\frac{\bar{m}_2 - g\nu}{a+\alpha+\nu} = \frac{\mu - \nu\bar{r} + (1+g)\bar{y} - g\nu}{2(a+\alpha+\nu)} \tag{22}$$

The first-order condition implies that $\bar{m}_t - g\nu > 0$. In order to analyze the effects of the parameters and the equilibrium inflation rate in period 2. Take partial derivatives of π_2^* with respect to a, α, g and $\bar{y}$. We have

$$\frac{\partial \pi_2^*}{\partial a} = \frac{\partial \pi_2^*}{\partial \alpha} = -\frac{\bar{m}_2 - g\nu}{2(a+\alpha+\nu)^2} < 0 \tag{23}$$

$$\frac{\partial \pi_2^*}{\partial g} = \frac{\bar{y} - \nu}{2(a+\alpha+\nu)} > 0 \tag{24}$$

$$\frac{\partial \pi_2^*}{\partial \bar{y}} = \frac{1+g}{2(a+\alpha+\nu)} > 0 \tag{25}$$

From the derivatives above, we have the following proposition

Proposition 2: *The incumbent government in period 2 will select the* ***unique*** *rate of inflation π_2^* which satisfies (22). The equilibrium inflation rate is positively related to the natural growth rate of output g as well as the output $\bar{y}$. However, it's negatively relevant to the parameters in adjustment and administration cost function, namely , a and α.*

Proposition 2 illustrates that the optimal rate of the inflation rises when the growth rate of the economy or the development of the country increases. The reason is that the incumbent doesn't need to consider the next period output and tax revenue. There are only two periods in our model. On the contrary, if the marginal cost of the deviation from the natural inflation increases, for example, a or α increases, then the optimal inflation rate will dereases. It's quite easy to understand since the deviation from the natural state is costly.

From (19) and (20), we have the difference of utility (*utility gap*) between the incumbent and the opposition in period t

$$\Delta W_t = W_t^I - W_t^O = \tau_0\bar{y}_t + (\pi_t + g)(\bar{m}_t - \nu\pi_t) - \frac{1}{2}\alpha\pi_t^2 \tag{26}$$

Differential the utility gap function with respect to a, α, g and $\bar{y}$. Substitute π_2^* into the utility gap function yields

$$\frac{\partial \Delta W_2^*}{\partial a} = Q\frac{\partial \pi_2^*}{\partial a} \tag{27}$$

$$\frac{\partial \Delta W_2^*}{\partial \alpha} = Q\frac{\partial \pi_2^*}{\partial \alpha} - \frac{1}{2}(\pi_2^*)^2 \tag{28}$$

$$\frac{\partial \Delta W_2^*}{\partial g} = Q\frac{\partial \pi_2^*}{\partial g} + R\bar{y} + \bar{m}_2 - \nu\pi_2^* \tag{29}$$

$$\frac{\partial \Delta W_2^*}{\partial \bar{y}} = Q\frac{\partial \pi_2^*}{\partial \bar{y}} + R(1+g) \tag{30}$$

where $Q = \left(\bar{m}_2 - g\nu - (\alpha + 2\nu)\pi_2^*\right)$,$R = (\pi_2^* + g + \tau_0)$. Since

$$\frac{\bar{m}_2 - g\nu}{\alpha + 2\nu} > \frac{\bar{m}_2 - g\nu}{2(a + \alpha + \nu)} = \pi_2^* \tag{31}$$

Then $\bar{m}_2 - g\nu - (\alpha + 2\nu)\pi_2^* > 0$, also

$$\frac{\bar{m}_2}{\nu} > \frac{\bar{m}_2 - g\nu}{\nu} > \frac{\bar{m}_2 - g\nu}{2(a + \alpha + \nu)} = \pi_2^* \tag{32}$$

Then $\bar{m}_2 - \nu\pi_2^* > 0$

With all above relation together we have

$$\frac{\partial \Delta W_2^*}{\partial \alpha} < \frac{\partial \Delta W_2^*}{\partial a} < 0, \qquad \frac{\partial \Delta W_2^*}{\partial g} > 0, \qquad \frac{\partial \Delta W_2^*}{\partial \bar{y}} > 0 \tag{33}$$

From (33) we have the following proposition

Proposition 3: *At equilibrium, higher growth rate of economy (g) and the natural output ($\bar{y}$) will broaden the utility gap between the incumbent and the opposition in period 2. On the contrary, larger parameters in cost function(namely a and α) will narrow the utility gap.*

This proposition conveys an idea that if the government gains more benefits from holding power, then the utility gap will be broadened. It's obvious since the incumbent can redistribute the tax revenues, and increases the incumbent group's utility. Conversely, the utility of the incumbent 2 will decreases. It will enlarge the utility gap and aggravate the inequality.

Now we turn to the inflation rate selection in period 1. For the incumbent government I_1, he faces the tradeoffs. The probability to stay in power decreases as he seeks for higher payoff in period 1. At the same time, the utility in period 2 is uncertain, and once the utility is realized, the utility has to be discounted (the utility in period 2 is less valuable). If I_1 stay in power in period 2, then he can redistribute the tax revenues.

In this problem, we have to use the backward induction method. The utility of the incumbent and the opposition in period 2 is given as former analysis. The incumbent in period 1 seek the optimal inflation rate in period 1 to maximize the life utility of his own group,

$$\max_{\pi_1} \quad W^{I_1} = W_1^{I_1} + \beta \left\{ \lambda(\pi_1) W_2^I \Big|_{\pi_2=\pi_2^*} + (1-\lambda(\pi_1)) W_2^O \Big|_{\pi_2=\pi_2^*} \right\} \tag{34}$$

It's just like (17). Differential (34) with respect to the rate of inflation in period 1 π_1 yields the first-order condition

$$\text{F.O.C:} \qquad \frac{\partial W_{I_1}}{\partial \pi_1} = \frac{\partial W_1^{I_1}}{\partial \pi_1} + \beta\lambda'(\pi_1)\left(W_2^I - W_2^O\right)\Big|_{\pi_2=\pi_2^*} = 0 \tag{35}$$

substitute $\pi_t = \pi_2^*$ into (26) yields the utility gap in equilibrium state

$$\Delta W_2^* = (W_2^I - W_2^O)\Big|_{\pi_2^*} = \tau_0(1+g)\bar{y} + (\pi_2^* + g)(\bar{m}_2 - \nu\pi_2^*) - \frac{1}{2}\alpha(\pi_2^*)^2 \tag{36}$$

Since π_1^* maximize the life-time utility of the incumbent in period 1, $\partial W_{I_1}/\partial\pi_1\Big|_{\pi_1=\pi_1^*} = 0$. Using (21) and the condition of utility maximization, we rewrite the first-order condition (35) as

$$(a+\alpha+\nu)\pi_1^* - \beta\lambda'(\pi_1^*)\Delta W_2^* = \frac{\bar{m}_1 - g\nu}{2} = \frac{\mu - \nu\bar{r} + \bar{y} - g\nu}{2} \tag{37}$$

In this section, we focus on the influence of growth rate and the output. In addition, we consider the effects of $\lambda'(\cdot)$ on the inflation rate in period 1 for I_1.

Differential (37) with respect to g and $\bar{y}$ at both sides and rearrange the result yields

$$\frac{\partial \pi_1^*}{\partial g} = \frac{\beta\lambda'(\pi_1^*)\frac{\partial \Delta W_2^*}{\partial g} - \frac{\nu}{2}}{(a+\alpha+\nu) - \beta\lambda''(\pi_1^*)\Delta W_2^*} \tag{38}$$

$$\frac{\partial \pi_1^*}{\partial \bar{y}} = \frac{\beta\lambda'(\pi_1^*)\frac{\partial \Delta W_2^*}{\partial \bar{y}} - \frac{1}{2}}{(a+\alpha+\nu) - \beta\lambda''(\pi_1^*)\Delta W_2^*} \tag{39}$$

We consider a special case. Since we cannot get the explicit expression of π_1^*, we cannot tell how large it is. But we know that the optimal inflation rate in period 1 exists and the value is positive. The special condition is

$$\lambda''(\pi_1^*) < \frac{a+\alpha+\nu}{\beta\Delta W_2^*} \tag{40}$$

From (33), we know that $\partial\Delta W_2^*/\partial g < 0$ and $\partial\Delta W_2^*/\partial\bar{y} < 0$. Since $\lambda'(\pi_1^*) < 0$, if the optimal inflation rate satisfies the condition (40), we have

$$\frac{\partial \pi_1^*}{\partial g} < 0 \qquad \frac{\partial \pi_1^*}{\partial \bar{y}} < 0 \tag{41}$$

Summering the views above, we get the next proposition

Proposition 4: *The incumbent government in period 1 will choose a rate of inflation which satisfies* (37). *If the optimal inflation rate satisfies the condition* (40)*, then the equilibrium inflation rate in period 1 is negatively relevant to the growth rate of economy (g) as well as the natural output ($\bar{y}$).*

The growth rate of economy and the natural output stand for the development of the economy (or the development of a country). The proposition says that under certain condition, the optimal inflation rate is low in more developed countries. The reason is quite simple. The incumbent government in period 1 wants to stay in power in period 2, since the incumbent have large benefits from redistribution.

That's exactly what happened in reality. In a democratic rich country, the inflation will be controlled to stay low level. Higher inflation will enlarge the high political risk.

At last, we check the relation between the stability of regime and the optimal inflation rate. The elasticity of regime stability with respect to inflation is given by

$$\varepsilon = \frac{(\ln \lambda)'}{(\ln \pi_1)'} = \lambda' \frac{\pi_1}{\lambda} \tag{42}$$

We want to check whether $\partial \pi_1 / \partial \varepsilon$ is positive or negative. Since

$$\frac{\partial \pi_1}{\partial \varepsilon} = \frac{1}{\partial \varepsilon / \partial \pi_1} \tag{43}$$

Then it's equivalent to check whether $\partial \varepsilon / \partial \pi_1$ is positive or negative

$$\frac{\partial \varepsilon}{\partial \pi_1} = \frac{\lambda'}{\lambda} - \frac{\pi_1}{\lambda^2} \left((\lambda')^2 - \lambda \lambda'' \right) \tag{44}$$

If the probability function has the property that $(\lambda')^2 - \lambda \lambda'' > 0$, then $\partial \varepsilon / \partial \pi_1$ is negative.

Proposition 5: *If the probability function $\lambda(\cdot)$ satisfies $(\lambda')^2 - \lambda \lambda'' > 0$ at equilibrium, then higher inflation elasticity of regime stability will result in lower inflation selection of the incumbent in period 1.*

Proposition 5 illustrates that under certain conditions, the higher political cost of inflation will lower the rate of inflation which the incumbent select to maximize his own group life-time utility. This is quite true since the inflation cost is high, the incumbent in period 1 have to give up more utility in period 2 which measured in expected utility. Thus the incumbent in period 1 will lower the inflation policy expecting higher expected utility in period 2.

3. Conclusion

Rational expectation gives the results in proposition 1. The expectation for inflation equals with the actual inflation. People will adjust their expectation if they are not equal. It is just like a game under perfect information. As a result, the actual output equals with the natural output, the growth rate of actual output $\tilde{g} = g$.

With equilibrium condition satisfied, we analysed the relationship between the inflation selection in two periods and the growth rate of output and the natural output. In period 2, the incumbent doesn't have to consider the next period's utility, hence the inflation rate will be relatively high. But in period 1, the incumbent have to consider the stability of regime. If the future utility is promising, the incumbent has stronger willing to stay in power in period 2. On the contrary, if the utility of the incumbent in period 2 is small, for example, the economy is less developed, and then the rate of inflation in period 1 will be relatively large. The relation between the inflation selection at equilibrium and the parameters are illustrated in proposition 2 and proposition 4.

The next problem is the utility gap between the incumbent and the opposition. Just as the proposition 3 says, in the equilibrium state, higher growth rate of economy and output will broaden the utility gap in period 2. On the contrary, larger parameters in cost function will narrow the utility gap.

The most important setting in our model is the probability that the incumbent in period stays in power in period 2. It will influence the expected utility in period largely. Proposition 5 illustrate that higher inflation elasticity of regime stability will result in lower inflation which is selected by the incumbent in period 1.

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